SOLUTION TO QUIZ 5

MATH 241

Calculate the Fourier series for $\sin(\frac{1}{2}x)$ on $[-\pi, \pi]$.

Proof. This is an odd function, so we will obtain a Fourier sine series.

$$\begin{aligned}
a_n &= \frac{2}{\pi}\int_0^\pi \sin\frac{1}{2}x\sin nx dx \\
&= \frac{1}{\pi}\int_0^\pi [\cos(\frac{1}{2}-n)x - \cos(\frac{1}{2}+n)x]dx \\
&= \frac{1}{\pi}[\frac{\sin(\frac{1}{2}-n)x}{\frac{1}{2}-n} - \frac{\sin(\frac{1}{2}+n)x}{\frac{1}{2}+n}]|_0^\pi \\
&= \frac{1}{\pi}[\frac{\sin(\frac{1}{2}-n)\pi}{\frac{1}{2}-n} - \frac{\sin(\frac{1}{2}+n)\pi}{\frac{1}{2}+n}] \\
&= \frac{1}{\pi}[\frac{(-1)^n}{\frac{1}{2}-n} - \frac{(-1)^n}{\frac{1}{2}+n}] \\
&= \frac{1}{\pi}\frac{2n(-1)^n}{\frac{1}{4}-n^2} \\
&= \frac{8n(-1)^n}{\pi(1-4N^2)}
\end{aligned}$$

Therefore $\sin(\frac{1}{2}x) = \sum_{n=1}^{\infty} \frac{8n(-1)^n}{\pi(1-4n^2)} \sin nx$. □